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VULNERABILITY OF COMPLEX NETWORKS AND URBAN TRANSPORTATION APPLICATIONS: A LITERATURE REVIEW

Laura Lotero Vélez[1]
Rafael Germán Hurtado Heredia[2]

ABSTRACT

In recent years, the interest of researchers and professionals from various disciplines in the analysis of social, biological, and artificial systems from the perspective of complex networks has grown considerably. Some of these networks, such as transportation networks, are part of critical infrastructure and are the basis of many human activities. One of the most important practical properties of complex networks is the ability to maintain some functions in the event of errors, failures, or attacks to their nodes or links; this property has been called robustness, resilience, or vulnerability by different authors. In this paper, we present a review of the literature on the concept of vulnerability in the formalism of complex networks and some perspectives of its application in the analysis of urban transportation networks. The purpose of this paper is to provide new insights to researchers and decision-makers in the analysis of mobility and transportation systems.

KEYWORDS: Complex networks; Robustness; Vulnerability; Urban transportation

VULNERABILIDAD DE REDES COMPLEJAS Y APLICACIONES AL TRANSPORTE URBANO: UNA REVISIÓN DE LA LITERATURA

RESUMEN

El interés de investigadores y profesionales de diversas disciplinas en el análisis de sistemas sociales, biológicos y artificiales desde la perspectiva de las redes complejas ha crecido notablemente en los últimos años. Algunas de estas redes son la base de muchas de las actividades del ser humano, como es el caso de las redes de transporte urbano que hacen parte de la infraestructura crítica. Una de las propiedades de mayor relevancia práctica de las redes complejas es su capacidad para mantener algunas funciones cuando ocurren fallas, errores o ataques a sus nodos o vínculos, la cual ha sido denominada robustez, resiliencia o vulnerabilidad por distintos autores. En este artículo se presenta una revisión de la literatura sobre el concepto de vulnerabilidad en el formalismo de las redes complejas y algunas aplicaciones al transporte urbano. El propósito de este artículo de revisión es el de dar a conocer a académicos y tomadores de decisión nuevos enfoques para el análisis del sistema de movilidad

PALABRAS CLAVE: redes complejas; robustez; vulnerabilidad; transporte urbano.

[1] Industrial engineer, Universidad Nacional de Colombia, Medellín. Master in Systems Engineering, Universidad Nacional de Colombia, Medellín. PhD (candidate) Engineering - Systems and Computer, Universidad Nacional de Colombia.

[2] Physical, Universidad Nacional de Colombia, Bogotá. PhD. in Physics, Universita degli Studi di Perugia, Italia. Professor Department of Physics, Universidad Nacional de Colombia, Bogotá.

Correspondence author Lotero-Vélez, L. (Laura): Universidad Nacional de Colombia sede Medellín, carrera 80 65 – 223, Bloque M8A – Oficina 402, / Teléfono: (57+4) 425 53 50 / Email: llotero0@unal.edu.co

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VULNERABILIDADE DE REDES COMPLEXAS E APLICAÇÕES AO TRANSPORTE URBANO: UMA REVISÃO DA LITERATURA

RESUMO

O interesse de investigadores e professionais de diversas disciplinas no análise de sistemas sociais, biológicos e artificiais desde a perspectiva das redes complexas cresceu notavelmente nos últimos anos. Algumas destas redes são a base de muitas atividades do ser humano, como é o caso das redes de transporte urbano que fazem parte da infraestrutura crítica. Uma de propriedades de maior relevância pratica das redes complexas é a sua capacidade para manter algumas funções quando acontecem falhas, erros o ataques a seus nodos o vínculos, a qual foi denominada robustez, resiliência e vulnerabilidade por distintos autores. Neste artigo apresentamos uma revisão da literatura sobre o conceito de vulnerabilidade no formalismo das redes complexas e algumas aplicações ao transporte urbano. A finalidade deste artigo de revisão é de dar a conhecer a acadêmicos e formadores de opinião novos enfoques para o análise do sistema de mobilidade.

PALAVRAS-CHAVE: Redes complexas; Robustez; Vulnerabilidade; Transporte urbano.

1. INTRODUCTION

The formal study of networks as a representation of social, biological, or artificial systems has experienced several stages, from the introduction of graphs to represent social systems in the 19th and early 20th centuries to the peak of complex networks in the 21st century. Conceptual contributions to the development of network analysis come from disciplines and specialties within economic, human, natural, and social sciences, mathematics, and engineering, framed in very diverse paradigms (Freeman, 2004; Newman, Barabasi, & Watts, 2006; Newman, 2010). In general, networks are represented in graphical terms, and theoretical and methodological constructions attend to the mathematical formulation of graph theory in order to explore its structure and function (Newman, 2010; Wasserman & Faust, 1994).

The study of probability distributions for connectivity properties in various large, real networks and the discovery of power laws in some of them (Barabási & Albert, 1999; Redner, 1998) paved the way for the study of complex systems, generally on a large scale and able to present emerging properties, using concepts and developments from statistical physics (Dorogovtsev & Mendes, 2003; Newman, Barabasi, & Watts, 2006; Newman, 2003, 2007, 2010). Today, this perspective is known as complex networks, which are characterized by having many degrees of freedom and possible configurations. One of the main characteristics of this perspective is that it allows for the establishment of a system's macroscopic properties based on the microscopic properties that involve the system's parts or elements.

Many complex networks are infrastructure networks critical to human beings (Setola & Porcellinis, 2009), including networks of aqueducts, sewers, irrigation, transportation, telecommunication, energy, etc., which can suffer failures, interruptions, random errors, congestion, or directed attacks. Events like terrorist attacks, nation-wide blackouts, and natural disasters have attracted the attention of various researchers who study the vulnerability or robustness of complex networks (Albert, Jeong, & Barabási, 2000; Cohen & Havlin, 2010; Latora & Marchiori, 2005; Nagurney & Qiang, 2011; Sydney, et al., 2010). For urban transportation networks, vulnerability analyses could be focused on analyzing what happens to the network in the event of traffic accidents, natural disasters that affect roadways because of repairs, protests, or city events, all of which imply, in terms of networks, the elimination of one or more connections within the network, which could affect connectivity and flows across it.

The purpose of this article is to explain the concepts of vulnerability or robustness that are found in the literature on complex networks and present some existing applications in the literature for urban transportation systems. This review is of interest to decision-makers, urban and transportation planners, and other actors. The article is organized in the following way: in section 2, we present general aspects of the phenomenology of complex networks as an area of research. Section 3 deals with the problem of vulnerability or robustness analysis in complex networks and contains some important results found in the literature. Section 4 explains the developments and applications of these analyses in urban transportation networks. Finally, section 5 presents the conclusions derived from this literature review.

2. COMPLEX NETWORKS

The perspective of complex networks was initiated in the late 1990s with wide conceptual and methodological support in the analysis of social networks and thanks to empirical and theoretical contributions made by statistical physics (Newman, 2010).

The mathematical bases of network analysis as an area of scientific research were established by Leonhard Euler (1736) with the solution of the Königsberg bridges problem, and, with it, the foundation of graph theory. The use of graphs to represent social systems led to the beginning of social network analysis during the first half of the 20th century (Freeman, 2004), when it was possible to draw analogies between the properties or phenomena present in real systems with some topological properties of the graphs that represent them. In the second half of the century, many of the concepts were specified or clarified, and mathematical analysis was formally introduced, including graph theory results and statistics (Erdös & Rényi, 1959; Freeman, 2004; Newman et al., 2006; Wasserman & Faust, 1994). Although graph theory, network analysis, and complex networks have been studied by diverse disciplines, the original problem of graph theory arose from a problem of urban transportation (Derrible & Kennedy, 2011).

In the development of the formalism of complex networks, which many consider to be a science in and of itself (Barabási, 2013; Vega-Redondo, 2007; Watts, 2004), biologists, sociologists, economists, and engineers from various areas have concentrated on collecting empirical information and on the phenomenology of diverse social, natural, and artificial systems. Mathematicians have made contributions to graph theory, and physicists have provided the theoretical bases for the definition of a concept that integrates empirical work, analysis, and modeling (Barabási, 2005). The contribution made by physicists to complex networks is the recognizing that, despite the apparent random nature of the system on a microscopic scale, there are macroscopic behaviors given by statistical laws that can be identified according to the topological characteristics of the graphs that represent the networks and that depend on the properties of the system's elements (Barabási, 2005).

The development of the formalism of complex networks has been concentrated on three branches of research (Newman, 2003), specifically: the network's statistical properties, which characterize its structure and topology (Bianconi, Pin, & Marsili, 2009; Boccaletti, et al., 2007; Borgatti, 2005; Costa, et al., 2007; Newman, 2003); network models that represent and help to understand the meaning of said properties (Barabási & Albert, 1999; Barrat, et al., 2004; Erdös & Rényi, 1959; Watts & Strogatz, 1998); and, finally, the analysis of dynamics and emerging behaviors within networks (Barrat, Barthélemy, & Vespignani, 2008; Boccaletti, et al., 2006; Dorogovtsev, et al., 2008; Nagurney & Qiang, 2007).

2.1. Models of Complex Networks

In the late 1950s, Hungarian mathematicians Paul Erdös and Alfred Rényi built a random network model to describe the evolution and some properties of communications networks (Erdös & Rényi, 1959). This model compares complexity with randomness: that which cannot be explained in simple terms due to its complexity is approached from the concept of randomness. The model proposed by Erdös & Rényi basically consists of randomly linking pairs of nodes to form the network and study the appearance of microscopic and macroscopic structures for different probabilities of link occurrence. The microscopic structures reference nodes and links individually, and the macroscopic structures reference the properties of the network as a whole, such as, for example, the degree distribution, the average number of steps between one node and another in the network, and the network's density, among others.

This model does not explain phenomena that are found in real social networks, as in the case of groups associated with the existence of communities united to each other by some weak links (Granovetter, 1973; Granovetter, 1983; Liu & Duff, 1972) and the occurrence of the so-called small-world phenomenon, which references the existence of short paths between two given elements in a network, although the network may be of considerable size (Milgram, 1967). In some ways, social relationships could not follow random laws, or at least not completely.

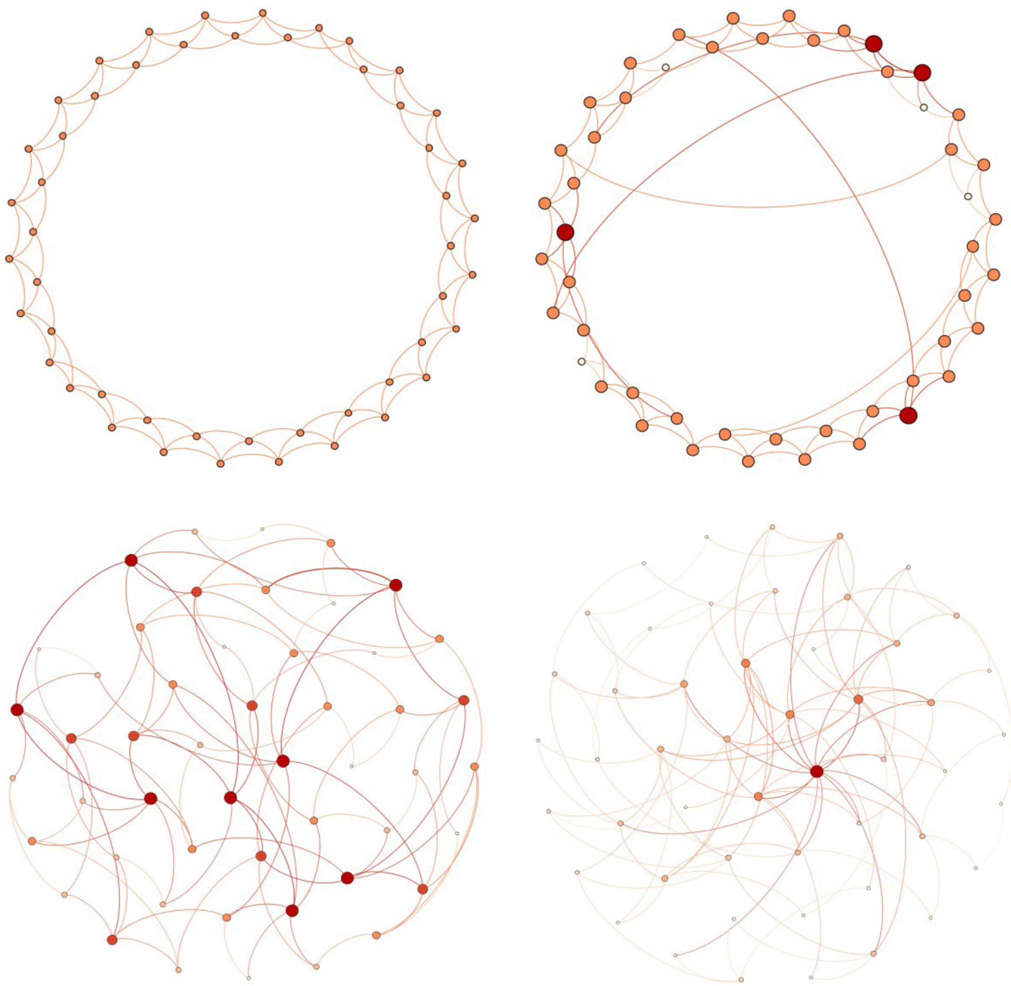
In the late 1990s, it was found that the phenomenon of grouping and the small-world phenomenon were not only present in social networks, but also in networks like those that describe the molecular interactions of some microorganisms and in electrical energy networks (Watts & Strogatz, 1998). The small-world model of networks proposed by Watts & Strogatz (1998) explains and describes networks in which it is possible to go from one given node in the network to another in a small average number of steps, conserving the grouping phenomenon, given by a high clustering coefficient in the network, characteristics that are present in small-world networks like those studied in sociology based on the Milgram experiment (Milgram, 1967; Travers & Milgram, 1969).

Also, Sidney Redner (1998), analyzing article citations in physics, and Barabási & Albert (1999), analyzing internet connectivity, discovered that these networks have properties that differ from those described by the models of networks known until that moment. They found a small amount of nodes with a large number of connections (hubs) which were not present in the random or small-world network models proposed by Watts & Strogatz (1998). In general, these networks present a power law for node degree distribution, that is, the number of links per node is large for a few nodes and small for many nodes. Therefore, these networks are called scale-free networks since the system observed at any scale will have the same connectivity principles. The scaling of a complex network has been recognized as a sign that the system is dynamic and subject to evolution or growth over time (Porta, Crucitti, & Latora, 2006) and that, in general, power laws are associated with critical and self-organizing phenomena.

Figure 1 shows different topologies for a network with 50 nodes, with an average number of 100 links. The first network is a lattice or regular network in which all the nodes have the same number of connections, in this case, four. The second is a small-world network according to the model proposed by Watts & Strogatz (1998), which, based on a regular network and re-linking probability for the links, allows for access from one point to another in the network in a relatively small number of steps. When the re-linking probability of the connections is equal to one, the resulting network is equivalent to a random network. The third is a random network according to the model proposed by Erdös & Renyí (1959), in which there are $n = 50$ nodes, $m = 100$ connections, and the probability that two nodes are connected, being a non-directed network, will be given by $p = 2m/(n^2-n)$, in which, in order to be comparable to the previous networks $p \approx 0{,}08$. The last is a scale-free network according to the model proposed by Barabási & Albert (1999), in which many of the nodes have few connections and a few nodes are highly connected. The color and size of the nodes in the networks in **Figure 1** are given by the degree, which is the property that indicates the number of connections that enter or exit the node.

Understanding the topology of complex networks is key to understanding the underlying complex systems. It is therefore necessary to establish the relationship between the network's topology and the system's dynamic, including the appearance of emergencies or collective phenomena, and generally the relationship to the network's function. In this sense, a vulnerability or robustness analysis of complex networks accounts for how the macroscopic status and the network's function change in the event of failures or attacks on its elements which produce changes in the system's microscopic structures. It is a relatively recent topic for research with open questions and a broad space for diverse applications.

Figure 1. Different topologies for a network with 50 nodes



3. VULNERABILITY OR ROBUSTNESS OF COMPLEX NETWORKS

There is no consensus regarding how to define the concept of vulnerability, which is also frequently associated with robustness or resilience. Definitions of vulnerability in different areas of study vary even within the same context (Ghedini & Costa Ribeiro, 2009; Jenelius, Petersen & Mattsson, 2006; McEntire, 2005; Newman, 2010).

Jenelius, Petersen, & Mattsson (2006) present definitions for vulnerability, trustworthiness, and risk in different contexts, including that of complex networks. They define vulnerability as a two-part concept: the first has to do with the probability that a dangerous event will occur, and the second, called exposition, has to do with the consequences of the event in a certain part of the network.

Holmgren (2006) defines vulnerability as the sensitivity of the system (its physical infrastructure in this case) to threats, risks, or disturbances that may arise. The author relates the term "vulnerability" with robustness and resilience, defining robustness as the system's ability to maintain its structure or functions intact or only slightly affected when it undergoes a disturbance, and resilience as the system's ability to recover after a disturbance. In this perspective, the concepts of robustness and resilience are complementary to that of vulnerability.

Wu et al. (2007) define the vulnerability of complex networks as the random failures or intentional attacks that affect the network's integrity and operation. On the other hand, Boccaletti et al. (2007) define the vulnerability of complex networks as the system's ability to maintain its functional performance in the face of random damages or malicious attacks. They use the term "vulnerability" as a concept to evaluate the stability and robustness of complex systems' global behaviors in the face of external disturbances.

Additionally, Ouyang and others (2009) hold that vulnerability is related to attacks and can be described as the reduction of the network's efficiency after an attack. According to these authors, vulnerability can be structural when only the network's topology is taken into account to measure structural efficiency, or functional when the network's levels of function are considered.

Nagurney & Qiang (2011) maintain that the analysis of vulnerability for complex networks deals with quantifying and evaluating the impact of removing a component from the network. The previous definition explicitly works with topology and the effect of removing elements on the network's performance. Another related definition is that of Gol'dshtein, Koganov, & Surdutovich (2004), who define network vulnerability as the relative drop in the network's performance after the removal of a vertex along with the connections that link it to other vertices. Latora & Marchiori (2005) present a similar definition.

The most common way to measure the vulnerability or robustness of a network is evaluating it after the elimination of one or more elements (whether these are nodes, connections, or a combination of both), in which said elimination may be random, simulating a failure, or } directed at an important element, simulating an attack (Albert, Jeong, & Barabási, 2000; Boccaletti, et al., 2007; Latora & Marchiori, 2005; Newman, 2003; Newman, 2010).

The directed removal of nodes generally begins by eliminating the most important nodes, which may be the most connected nodes, or hubs. A node's importance is determined by various factors associated with its connectivity and that of the network as a whole. Some of the properties of nodes are the centrality measurements of node degree, defined as the number of connections a node has; the centrality of intermediation, defined according to the number of geodesic paths or short paths between pairs of nodes that pass through the node of interest; the centrality of proximity, given by the inverse of the sum of the distances from the node to the remaining nodes; the cohesion coefficients, given by the connectivity between neighboring nodes and the geodesic lengths (Newman, 2007, 2010; Wasserman & Faust, 1994).

In complex network vulnerability and robustness analyses, the results found by Albert, Jeong, & Barabási (2000) mark a starting point; they found that some aspects of said properties depend on networks' topology, particularly on node degree distribution. So, scale-free networks are considered to be robust in the face of errors or random failures, while they are vulnerable to attacks directed at the most connected nodes. In contrast, random networks are robust in the face of attacks directed at the most connected nodes.

On the other hand, the results found by Newman (2002) show that networks with selective configuration in their degree distributions are more robust when highly connected nodes are removed. A selective configuration appears when the nodes with the largest number of connections are connected with other highly-connected nodes. This behavior can be observed in social networks, while technological and biological networks tend not to have selective configurations. These results suggest that social networks are less vulnerable than biological or artificial networks to attacks on their most important nodes.

In the literature on complex network vulnerability, different methods for analyzing the effects of removing an element from the network are proposed, such as establishing how much the geodesic paths change (Boccaletti, et al., 2007; Latora & Marchiori, 2005; Mishkovski, Biey, & Kocarev, 2011), evaluating how much the network is fragmented (Albert, Jeong, & Barabási, 2000; Newman, 2002; Wu, et al., 2007), or evaluating changes in the flows within the network (Nagurney & Qiang, 2007). However, there is no agreement on an analysis methodology that systematically takes into account the changes in the network's topology in order to measure and study a network's vulnerability in the event of different possible occurrences.

4. DEVELOPMENTS AND APPLICATIONS IN URBAN TRANSPORTATION

Systems of transportation and urban mobility are complex systems made up of a large number of interconnected elements that also show emerging, nonlinear collective behaviors (Amaral & Ottino, 2004). Therefore, a plausible approach for analysis of these systems could be the use of conceptual and analytical tools from the science of complexity, including that of complex networks, which allow for direct mapping of the system's elements in nodes and links between them. One of the most important aspects of complex network analysis is that, based on their topology, it is possible to determine some of their dynamic properties, thereby offering new methodological tools for a better understanding of phenomena specific to urban transportation, such as traffic congestion.

Despite the fact that a significant number of different kinds of networks have been studied by the science of complex networks, the study of urban transportation networks has been limited, and the approaches that can be found in the literature have mainly been proposed by physicists, not by engineers or transportation planners (Derrible & Kennedy, 2011).

4.1. Urban Transportation Networks

The use of graph theory in urban transportation systems arose between the 1950s and the 1970s in order to predict regional economic impacts on interstate highway systems in the United Stated (Derrible & Kennedy, 2011). With the development of computation, models of transportation networks became more information intensive, and four-stage models, widely used in transportation planning, appeared (Ortúzar & Willumsen, 2011). These models divide the problem of urban transportation planning into models of demand and of supply. The demand models estimate the number of trips from a starting point (trip generation) to different destination areas (trip distribution) for different available modes of transportation (mode partition).

The supply models divide or assign the trips generated in the demand models on the roadway network with the goal of balance between supply and demand (Ortúzar & Willumsen, 2011).

Based on this separation, two types of subsystems are defined: transportation supply and demand, which in turn can be represented by two different networks: a mobility network that represents the origin-destination matrix from the demand models and a network of roads along which the trips are assigned in the supply models. Both are directed networks since the connections between nodes have a defined direction, and they are heavy networks because the connections have different flows, which in the case of the mobility network are the number of trips between the origin-destination areas, and in the case of the roadway network, they can be the road's capacity, its length, and trip times, among other attributes.

The vulnerability and robustness analyses for urban mobility networks have been focused primarily on analyzing problems of spreading illnesses or information based on flows between origin and destination zones (Balcan, et al., 2009; Belik, Geisel, & Brockmann, 2011; González, Hidalgo, & Barabási, 2008) and indirectly on analyzing problems of accessibility from an urban planning context (de Montis, Caschili, & Chessa, 2011; Caschili & de Montis, 2013), from a social inclusion perspective (Hernández, 2012), or from a geographical perspective (Rodríguez-Nuñez, 2012). In the case of urban transport roadway networks, which can be the entire roadway grid or a sub-graph of it (for example, a subway or bus network), analyses have been focused on the effects of a failure or an attack on the roadway infrastructure and its consequences.

The following section reviews studies analyzing the vulnerability of urban transportation roadway networks, both from a traditional transportation analysis perspective and from the perspective of complex networks.

4.2. Vulnerability and Robustness of Urban Transportation Roadway Networks

In the analysis of vulnerability and robustness of roadway networks from traditional transportation analysis, equilibrium or optimization models that follow the principles of traffic assignment proposed by Wardrop (1952) are used to measure the effects of removing an element from the transportation network on the trip's general costs.

Jenelius, Petersen, & Mattsson (2006) introduce the concepts of the importance of the stretch of roadway and the location's exposure based on the increase in general trip costs when a roadway is closed. These concepts were applied to the vulnerability analysis of the roadway network in northern Sweden. The study proposes two focuses: that of equal opportunities and that of social efficiency. In the first case, all the roadways have the same weight in the analysis, while in the second, the roadways with a greater trip demand are more important. When calculating the function of general trip costs (time), the stretches of roadway are weighted by demand, meaning that stretches with a greater weight produce a greater increase in the cost function. Considerations on user behavior in the transportation system follow the user equilibrium principle (Wardrop, 1952).

Similarly, Nagurney & Qiang (2007) propose a measurement of efficiency for the network that identifies its most important, elements taking into account the user equilibrium principle in the network. Therefore, this measurement involves distances and also costs, flows, and user behavior within the system. They present applications to an electrical energy distribution network, a transportation network, and the case of the Braess paradox, which establishes that increasing the capacity of a network when the agents egotistically choose their route can, in some cases, reduce the network's overall performance (Braess, Nagurney y Wakolbinger, 2005).

Eustace, Russell, & Dean (2012) use a robustness analysis to decide which stretches of roadway in the network should be given priority status in the development of transportation plans. The measure of robustness for the stretch of roadway is based on how many times it has been congested in different situations. Here, congestion is measured as the volume-capacity relationship resulting from a traffic assignment model.

The disadvantage of using equilibrium or optimization models for vulnerability or robustness analysis of transportation networks is the computational complexity when considering the effects of congestion since this creates a nonlinear combination model which increases the complexity as the size of the network increases (Patriksson, 1994; Lotero, Jaramillo, & Rave, 2013).

On the other hand, from the perspective of complex networks, Latora & Marchiori (2005) define a measurement of network efficiency that identifies critical components, that is, the network's most vulnerable elements, and estimates the importance of an attack or failure on the network as the relative drop in the network's performance in terms of its efficiency. Thus, the vulnerability of the network or physical infrastructure is given by the relationship between the network's performance after the worst attack or failure and its initial performance. In this study, the authors present the vulnerability analysis of Boston's metro system and identify the most critical stations and lines. A similar approach, using the same efficiency indicator, was used by Chang and collaborators (2006) to analyze and compare the metro systems in Seoul, Tokyo, Boston, and Beijing. They also propose the network's response to disconnection between elements as an analysis of robustness through the analysis of triangular sub-graph formation; a large number of triangles in the network offers more alternatives in the event of a failure or attack on the network.

Han & Liu (2009) take ten metro networks in China as a basis for their analysis and consider both errors and attacks for their vulnerability analysis. In the analysis of tolerance to errors, they randomly remove nodes from the network, and in the analysis of vulnerability to attacks, they use four methods of node elimination: according to their initial node degree, according to their initial intermediation value, according to the node degree recalculated after the removals, and according to the recalculated value of intermediation. Of these four methods, the one that affected the network the most was that based on recalculated intermediation. Among other results, the authors find that despite the fact that the evaluated networks are not scale-free, they fulfill the property described for said networks of being robust in the event of random node removal and vulnerable to attacks on important nodes.

Derrible & Kennedy (2010) study 33 metro systems and propose a robustness analysis, understanding robustness to be the existence of alternative routes for users in the event of accidents or failures. The measure of robustness depends on the number of cycles present in the graph that represents the system and on the connections' tendency to fail. The results of this study show that the network is more robust the more united it is. Likewise, they show that the robustness of these metro systems depends on the network's size, the scaling coefficient (whether the network obeys a power law), the possibilities of transfers between lines, and cohesion coefficients.

Most approaches to vulnerability analyses in urban transportation networks using complex networks have been focused on metro networks whose size is considerably smaller than a transportation network that takes the entire roadway grid into account.

5. CONCLUSIONS

This article presents a literature review for vulnerability analysis of complex networks with cases of application to urban transport which, as far as the authors are aware, does not exist to date and could be a tool to complement decision-making processes in urban and transportation planning on the local, national, and international level.

The formalism of complex networks has been used recently to describe and analyze multiple social, biological, and artificial systems, including systems of mobility and urban transportation. Currently, there is no consensus in this perspective regarding the concepts that support evaluation of vulnerability in complex networks or regarding the most appropriate analysis methodologies. There is also no practice of applying this type of analysis to systems of practical interest. It is therefore necessary to make methodological clarifications that facilitate spreading this perspective to support technical, planning, political, and decision-making processes.

In the case of urban transportation, we have found very recent applications of a limited number, despite the fact that these systems have a natural representation in graphic terms. Applications have been found to metro networks in North America, Europe, and Asia, but no applications have been found for complete roadway networks. Applications of this formalism to vulnerability analyses for urban transportation systems were not found at the local, national, international, or even Latin American level, which indicates that it is necessary to spread the conceptual characteristics and methodologies that support this perspective, as well as the practical advantages of its application, among researchers, technicians, and decision-makers.

REFERENCES

Albert, R.; Jeong, H. y Barabási, A. L. (2000). Error and Attack Tolerance of Complex Networks. *Nature*, 406(6794), pp. 378-82.

Amaral, L. A. y Ottino, J. M. (2004). Complex Networks: Augmenting the Framework for the Study of Complex Systems. *European Physical Journal B,* 38(2), pp. 147-162.

Balcan, D.; Colizza, V.; Gonçalves, B. H.; Hao, *et al*. (2009). Multiscale Mobility Networks and the Spatial Spreading of Infectious Diseases. *Proceedings of the National Academy of Sciences*, *106*(51), pp. 21484-21489.

Barabási, A. L. (2005). Taming Complexity. *Nature Physics*, 1(2), pp. 68-70.

Barabási, A. L. y Albert, R. (1999). Emergence of Scaling in Random Networks. *Science*, 286, October, pp. 509-512.

Barabási, A. L. (2013). Network Science. *Philosophical Transactions. Series A, Mathematical, Physical, and Engineering Sciences*, 371(1987), pp. 20120375.

Barrat, A.; Barthélemy, M.; Vespignani, A. (2008). *Dynamical Processes on Complex Networks*. New York: Cambridge University Press, 368p.

Barrat, A.; Barthélemy, M.; Pastor-Satorras, R. y Vespignani, A. (2004). The Architecture of Complex Weighted Networks. *Proceedings of the National Academy of Sciences of the United States of America*, 101(11), pp. 3747-52.

Belik, V.; Geisel, T. y Brockmann, D. (2011). Natural Human Mobility Patterns and Spatial Spread of Infectious Diseases. *Physical Review X*, 1(1), pp. 1-5.

Bianconi, G.; Pin, P. y Marsili, M. (2009). Assessing the Relevance of Node Features for Network Structure. *Proceedings of the National Academy of Sciences*, 106(28), pp. 11433-11438

Boccaletti, Stefano; Buldú, Javier; Criado, Regino. *et al.* (2007). Multiscale Vulnerability of Complex Networks. *Chaos*, 17(4), pp. 043110.

Boccaletti, S.; Latora, V.; Moreno, Y., *et al*. (2006). Complex Networks: Structure and Dynamics. *Physics Reports*, 424(4-5), pp. 175-308.

Borgatti, S. (2005). Centrality and Network Flow. *Social Networks*, 27(1), pp. 55-71.

Braess, D.; Nagurney, A. y Wakolbinger, T. (2005). On a Paradox of Traffic Planning. *Transportation Science*, 39(4), pp. 446-450.

Caschili, S. y de Montis, A. (2013). Accessibility and Complex Network Analysis of the U.S. Commuting System. *Cities,* 30(February), pp. 4-17.

Chang, K.; Kim, K.; Oshima, H. y Yoon, S. (2006). Subway Networks in Cities. *Journal of the Korean Physical Society*, 48, pp. 143–145.

Cohen, R. y Havlin, S. (2010). *Complex Networks: Structure, Robustness and Function*. Cambridge: Cambridge University Press, 2010. 248p.

Costa, L. da F.; Rodrigues, F.; Travieso, G., *et al*. (2007). Characterization of Complex Networks: A Survey of Measurements. *Advances in Physics*, 56(1), pp. 167-242.

De Montis, A.; Caschili, S. y Chessa, A. (2011). Spatial Complex Network Analysis and Accessibility Indicators: the Case of Municipal Commuting in Sardinia, Italy. *European Journal of Transport and Infrastructure Research*, 11(4), pp. 405-419

Derrible, S. y Kennedy, C. (2010). The Complexity and Robustness of Metro Networks. *Physica A: Statistical Mechanics and its Applications*, 389(17), pp. 3678-3691.

Derrible, S. y Kennedy, C. (2011). Applications of Graph Theory and Network Science to Transit Network Design. *Transport Reviews: A Transnational Transdisciplinary Journal,* 31(4), pp. 495-519.

Dorogovtsev, S. y Mendes, J. F. (2003) *Evolution of Networks: From Biological Nets to the Internet and WWW.* New York, USA: Oxford University Press. 280p

Dorogovtsev, S.; Goltsev, A. y Mendes, J. F. (2008). Critical Phenomena in Complex Networks. *Reviews of Modern Physics*, 80(4), pp. 1275- 1335.

Erdös, P. y Rényi, A. (1959). On Random Graphs, I. *Publicationes Mathematicae,* 6(December), pp. 290–297.

Euler, L. (1736). Solutio problematis ad geometriam situs pertinentis. *Commentarii Academiae Scientiarum Imperialis Petropolitanae*, 8, pp. 128–140.

Eustace, D.; Russell, E. y Dean, L. (2012). Incorporating Robustness Analysis into Urban Transportation Planning Process. En: Levinson D, Liu H y Bell M (eds) *Network reliability in practice*. Springer, New York, pp. 97-114

Freeman, L. (2004). *The Development of Social Network Analysis: A Study in the Sociology of Science*. Vancouver: Empirical Press, 2004. 208p

Ghedini, C. y Costa-Ribeiro, C. (2009). A Framework for Vulnerability Management in Complex Networks. *2009 International Conference on Ultra Modern Telecommunications & Workshops*, pp. 1-8. IEEE.

Gol'dshtein, V.; Koganov, G. y Surdutovich, G. Vulnerability and Hierarchy of Complex Networks. [Documento online]. *Los Álamos National Laboratories Repository*. 2004: Available on: <http://arXiv.org/abs/cond-mat/0409298>. [Consulted March 7 2012].

González, M.; Hidalgo, C., Barabási, A. L. (2008). Understanding Individual Human Mobility Patterns. *Nature*, 453(7196), pp. 779-782.

Granovetter, M. (1973). The Strength of Weak Ties. *American Journal of Sociology*, 78(6), pp. 1360-1380.

Granovetter, M. (1983). The Strength of Weak Ties: A Network Theory Revisited. *Sociological Theory*, 1, pp. 201-233.

Han, C. y Liu, L.. (2009). Topological Vulnerability of Subway Networks in China. *2009 International Conference on Management and Service Science*, pp. 1-4.

Hernández, D.. (2012). Activos y estructuras de oportunidades de movilidad: Una propuesta analítica para el estudio de la accesibilidad por transporte público, el bienestar y la equidad. *EURE (Santiago)*, 38(115), pp. 117–135.

Holmgren, A. (2006). Using Graph Models to Analyze the Vulnerability of Electric Power Networks. *Risk Analysis: An Official Publication of the Society for Risk Analysis*, 26(4), pp. 955-969.

Jenelius, E.; Petersen, T. y Mattsson, L. G. (2006). Importance and Exposure in Road Network Vulnerability Analysis. *Transportation Research Part A: Policy and Practice*, 40(7), pp. 537-560.

Latora, V. y Marchiori, M. (2005). Vulnerability and Protection of Infrastructure Networks. *Physical Review E*, 71(1), pp. 1-4.

Liu, W. y Duff, R. (1972). The Strength of Weak Ties. *The Public Opinion Quarterly*, 36(3), pp. 361-366.

Lotero, L.; Jaramillo, P., y Rave, C. C. (2013). A Heuristic Decomposition Method for Large-Scale Traffic Assignment: Aburra Valley Case Study. *Revista Facultad de Ingeniería Universidad de Antioquia*, 68, pp. 187–197.

McEntire, D. (2005). Why Vulnerability Matters: Exploring the Merit of an Inclusive Disaster Reduction

Concept. *Disaster Prevention and Management*, 14(2), pp. 206-222.

Milgram, S. (1967). The Small World Problem. *Psychology Today*, 2(1), pp. 60–67.

Mishkovski, I.; Biey, M. y Kocarev, L. (2011). Vulnerability of Complex Networks. *Communications in Nonlinear Science and Numerical Simulation*, 16(1), pp. 341-349.

Nagurney, A. y Qiang, Q. (2007). A Network Efficiency Measure with Application to Critical Infrastructure Networks. *Journal of Global Optimization*, 40(1-3), pp. 261-275.

Nagurney, A. y Qiang, Qiang (2011). Fragile Networks: Identifying Vulnerabilities and Synergies in an Uncertain Age. *International Transactions in Operational Research*, 19(1-2), pp. 123-160.

Newman, M. (2002). Assortative Mixing in Networks. *Physical Review Letters*, 89(20), pp. 1-4.

Newman, M. (2003). The Structure and Function of Complex Networks. *SIAM Review*, 45(2), pp. 167-256.

Newman, M.; Barabási, A. y Watts, D. *The Structure and Dynamics of Networks*. Princeton, New Jersey, USA: Princeton University Press, 2006. 592p.

Newman, M. (2007). The Mathematics of Networks. *The new palgrave encyclopedia of economics*, pp. 1-12.

Newman, M. *Networks: an introduction*. New York, New York, USA: Oxford University Press, 2010. 784p.

Ortúzar, J. de D. y Willumsen, L. *Modelling transport (4th ed.)* Chichester: John Wiley & Sons, Ltd. 2011. 606p.

Ouyang, Min; Hong, Liu; Mao, Zi-Jun; *et al*. (2009). A Methodological Approach to Analyze Vulnerability of Interdependent Infrastructures. *Simulation Modelling Practice and Theory*, 17(5), pp. 817-828.

Patriksson, M. *The traffic assignment problem, models and methods*. Utrecht: VSP. 1994. 222p.

Porta, S.; Crucitti, P. y Latora, V. (2006). The Network Analysis of Urban Streets: A Dual Approach. *Physica A: Statistical Mechanics and its Applications*, 369(2), pp. 853-866.

Redner, S. (1998). How Popular is your Paper? An Empirical Study of the Citation Distribution. *The European Physical Journal B-Condensed Matter and Complex Systems*, 4(2), pp. 131–134.

Rodríguez-Núñez, E. (2012). Estudio de la vulnerabilidad de redes de transporte desde una perspectiva geográfica. *Revista de Estudios Andaluces*, 29, pp. 59-82.

Setola, R. y Porcellinis, S. (2009). *Complex Networks and Critical Infrastructures*. En: Chiuso, A.; Fortuna, L.; Frasca, M.; Rizzo, A.; Schenato, L. y S. Zampieri (Eds.) Modelling, Estimation and Control of Networked Complex Systems. Berlin, Heidelberg: Springer Berlin Heidelberg, pp. 91–106.

Sydney, A.; Scoglio, C.; Youssef, M. y Schumm, P. (2010). Characterizing the Robustness of Complex Networks. *International Journal of Internet Technology and Secured Transactions*, 2(3-4), pp. 291-320.

Travers, J. y Milgram, S. (1969). An Experimental Study of the Small World Problem. *Sociometry*, 32(4), pp. 425-443.

Vega-Redondo, F. *Complex Social Networks*. Cambridge University Press, 2007. 294p.

Wardrop, John. (1952). Some Theoretical Aspects of Road Traffic Tesearch. *Proceedings of the Institute of Civil Engineers*, 1, pp. 325-378.

Wasserman, S. y Faust, K. *Social network analysis: Methods and applications*. Cambridge university press, 1994. 857p.

Watts, D. y Strogatz, S. (1998). Collective Dynamics of Small-World Networks. *Nature*, 393(6684), pp. 440-442.

Watts, D. (2004). The "New" Science of Networks. *Annual Review of Sociology*, 30, pp. 243–270.

Wu, Jun; Deng, Hong-Zhong; Tan, Yue-Jin y Zhu, Da-Zhi. (2007). Vulnerability of Complex Networks Under Intentional Attack with Incomplete Information. *Journal of Physics A: Mathematical and Theoretical*, 40(11), pp. 2665-2671.

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